The Monte Carlo method — a motivational example

The Monte Carlo method has gained tremendous popularity in a variety of industries because of its noticeable amount of success in the estimation of solutions to high-dimensional problems.
One can understand the basic need for and principles underlying the Monte Carlo method through the following example: suppose on a sheet of paper we draw some shapes like that in Figure 1. Next, suppose we want to know the area covered by these shapes as a percentage of the area of the drawing region. How do we do this? There are likely many ways. The Monte Carlo method would estimate this area in a probabilistic manner.

Figure 1: A random drawing — we want to estimate the area of the shaded region as a percentage of the canvas on which it is drawn

Suppose now, that we have access to darts with very fine tips. Hanging the artwork on a wall and standing at a reasonable distance so that our aim would be worthless, we throw darts at the paper. After throwing a good many of them, we count the number of darts that landed inside the drawn region and divide it by the total darts thrown to give us an estimate for the area as a percentage of the area of the paper! This is, effectively, the Monte Carlo method.
On a computer, we can simulate this act of dart throwing. What this amounts to is 1) choosing points at random from a uniform distribution on some rectangle in the x-y plane that bounds the objects whose area is to be estimated, 2) counting the number of points that landed within the boundary of the objects, and 3) computing the ratio of the points that landed within the boundary of the objects to the total number of random points used. This gives an estimate for the area covered by the objects as a percentage of the bounding rectangle. The more random points used, the better the estimate. Figure 2 shows an example of this.

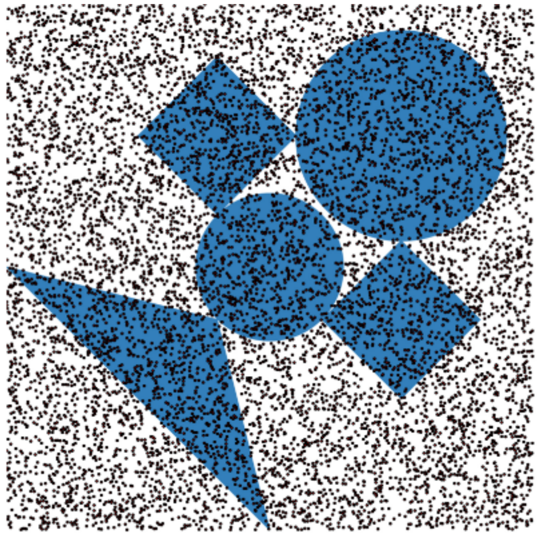

Figure 2 A Monte Carlo estimate for the area encompassed by the shaded region is 36.6% of the bounding rectangle — 3,660 of the 10,000 points used landed within the shaded region

In the example of Figure 2, the shapes were all distinct and hence one could compute the area using known formulas for area. However, when the regions become more complicated, analytical solutions are not always possible. Figures 3 and 4 show how we can use the Monte Carlo method even when the region begins to get more complicated. This technique of “throwing darts” is known as the “Hit or Miss Monte Carlo method” [Reuven Y. Rubinstein. *Simulation and the Monte Carlo Method* John Wiley & Sons, New York, 1981.].

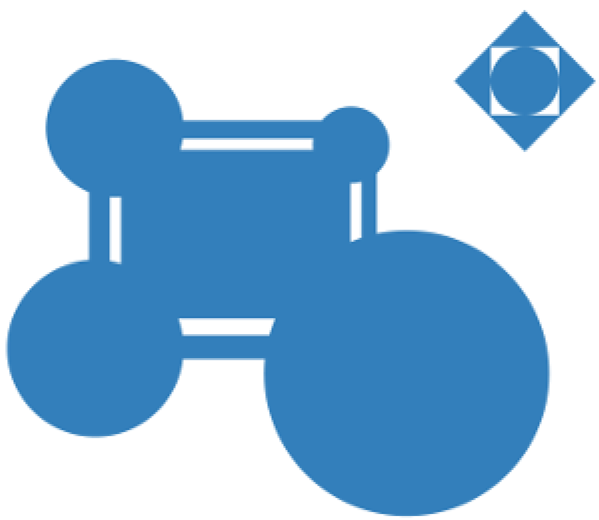

Figure 3 A more complicated region for which direct methods to compute the area may not work very easily

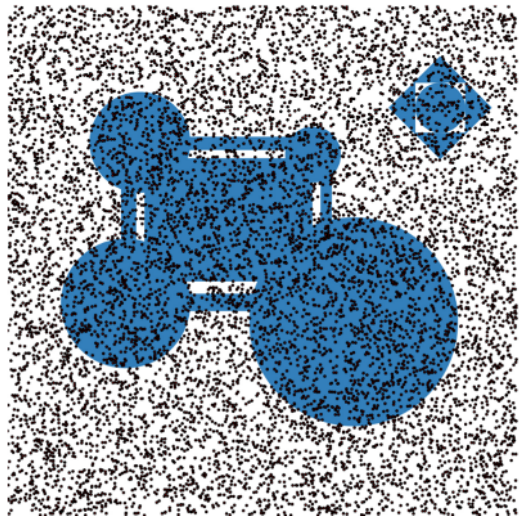

Figure 4 A Monte Carlo estimate for the area encompassed by the shaded region is 31.35% of the bounding rectangle — 3,135 of the 10,000 points used landed within the shaded region